Int J Theor Phys (2008) 47: 2453–2463
DOI 10.1007/s10773-008-9679-0

Anomalies and Hawking Radiation of NUT-Kerr-Newman de Sitter Black Hole

Kai Lin · Shi Wu Chen · Shu Zheng Yang

Received: 17 November 2007 / Accepted: 28 January 2008 / Published online: 12 February 2008

Abstract Hawking radiation of NUT-Kerr-Newman de Sitter black hole is studied via anomalous point of view in this paper. The results show that the charged current and energy-momentum tensor fluxes, to restore gauge invariance and general coordinate covariance at the quantum level in the effective field theory, are exactly equal to those of Hawking radiation from the event horizon (EH) and the cosmological horizon (CH) of NUT-Kerr-Newman de Sitter black hole, respectively.

Keywords Gauge anomaly · Gravitational anomaly · Hawking radiation · NUT-Kerr-Newman de Sitter black hole

1 Introduction

Black holes are interesting heavenly bodies, though their existence has not been recognized. More and more properties of black holes have been researched [1–10] since Hawing proved that black hole has thermal radiation in 1974 [11]. Now, people relate black hole physics with thermodynamics, and think that black hole physics have four laws as well as thermodynamics. On the other hand, the information loss of black hole is still a big problem. But, we can confirm that the basis of four laws and information problem of black hole is Hawking radiation, which is conclusion of quantum effect. The ways of proving Hawking radiation are so beautiful that they almost used the most advanced theory then. Recently, Robinson and Wilczek proved Hawking radiation from Schwarzschild black hole via anomaly [12]. The theory is established on the point of view of $(1+1)$-dimensional effective theory and anomalies cancellations. S. Iso, K. Murata, S.Q. Wu and Q.Q. Jiang etc. have researched several stationary black holes with the theory [13–20], and have obtained satisfactory results. The Hawking radiation from NUT-Kerr-Newman de Sitter black hole has been studied with some other ways [21]. But, in this paper, we will research this black hole via gauge

K. Lin (✉) · S.W. Chen · S.Z. Yang
College of Physics and Electric Information, China West Normal University, Nanchong, Sichuan 637002, People's Republic of China
e-mail: lk314159@126.com

anomaly and gravitational anomaly point of view. The procedures are as follows. Firstly, a $(1+1)$-dimensional effective theory in NUT-Kerr-Newman de Sitter spacetime will be founded by the dimension reduction technique. Next, we just consider outgoing modes at event horizon (or ingoing modes at cosmological horizon) and find an effective field theory. Finally, a covariant current is introduced to cancel the anomalies. We can prove the current fluxes are exactly equal to those of Hawking radiation.

2 The 2-Dimensional Theory of Nut-Kerr-Newman de Sitter Black Hole

In this section, the NUT-Kerr-Newman de Sitter black hole is reviewed and physics near the horizons is described by an infinite collection of $(1+1)$-dimensional fields.

Considering the metric of the black hole in Boyer-Lindquist coordinate system [22]

$$\begin{aligned} ds^2 = &-\frac{\Delta_r - \Delta_\theta a^2 \sin^2\theta}{\rho^2 \Xi^2} dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \\ &- 2\frac{\Delta_\theta a(r^2+a^2)\sin^2\theta - \Delta_r(a - \frac{(n-a\cos\theta)^2}{a})}{\rho^2 \Xi^2} dt d\phi \\ &+ \frac{\Delta_\theta (r^2-a^2)^2 \sin^2\theta - \Delta_r (a - \frac{(n-a\cos\theta)^2}{a})^2}{\rho^2 \Xi^2} d\phi^2 \end{aligned} \tag{1}$$

where

$$\begin{aligned} &\rho^2 = r^2 + \left(n - a^2\cos^2\theta\right)^2, \\ &\Delta_r = \left(r^2 + a^2 - n^2\right)\left(1 - \frac{1}{3}\Lambda r^2\right) - 2Mr + Q^2, \\ &\Delta_\theta = 1 + \frac{1}{3}\Lambda\,(n - a\cos\theta)^2, \quad \Xi = 1 + \frac{1}{3}\Lambda\left(a^2 - n^2\right) \end{aligned} \tag{2}$$

in which, *n, M, a, Q* are NUT parameter, mass, angular momentum and charge of black hole, respectively. Next, we have

$$g = \det(g_\mu \nu) = -\Xi^{-4}\rho^4 \sin^2\theta, \tag{3}$$

$$\begin{aligned} \partial_s^2 = &-\frac{[\Delta_\theta(r^2-a^2)^2\sin^2\theta - \Delta_r(a - \frac{(n-a\cos\theta)^2}{a})^2]}{\rho^2\Delta_\theta\Delta_r\sin^2\theta\,\Xi^{-2}}\partial_t^2 \\ &+ \frac{\Delta_r}{\rho^2}\partial_r^2 + \frac{\Delta_\theta}{\rho^2}\partial_\theta^2 + \frac{\Xi^2[\Delta_r - \Delta_\theta a^2\sin^2\theta]}{\rho^2\Delta_\theta\Delta_r\sin^2\theta}\partial_\phi^2 \\ &- \frac{2[\Delta_\theta(r^2-a^2)a\sin^2\theta - \Delta_r(a - \frac{(n-a\cos\theta)^2}{a})]}{\rho^2\Delta_\theta\Delta_r\sin^2\theta\,\Xi^{-2}}\partial_t\partial_\phi. \end{aligned} \tag{4}$$

From the line element (1), we know that the black hole is charged, rotational and axial symmetry. Other than Kerr-Newman black hole, the black hole has CH and NUT parameter, so the black hole can be understood as a generalization of Kerr-Newman black hole. From the null hypersurface equation

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu} = 0. \tag{5}$$

We can obtain a negative root denoted by r_- (but it is meaningless, we should abandon it) and three positive roots, r_h (the event horizon), r_c (the cosmological horizon) and r_0 (the inner horizon, IH). But the roots are different from each other. What we are interested in are r_h and r_c. The electromagnetic potential can be expressed as

$$A_\mu = (A_t, 0, 0, A_\phi) \tag{6}$$

where $A_t = -\frac{Qr}{\rho^2\Xi}$, $A_\phi = -\frac{Qra\sin^2\theta}{\rho^2\Xi}$. Now, we show that the scalar field theory which describes the line element (1) could be reduced to $(1+1)$-dimensional theory. The action of the scalar field is

$$\begin{aligned}
S[\Phi] &= \frac{1}{2}\int d^4x\sqrt{-g}\Phi\nabla^2\Phi \\
&= \frac{1}{2}\int dtdrd\theta d\phi\frac{\rho^2\sin\theta}{\Xi^2}\Phi\Bigg[\frac{-\Delta_\theta(r^2+a^2)^2\sin^2\theta}{\Xi^{-2}\rho^2\Delta_\theta\Delta_r\sin^2\theta}\left(\frac{\partial}{\partial t}+\frac{ieQr}{\rho^2\Xi}\right)^2 \\
&\quad + \frac{\Delta_r(a-(n-\cos\theta)^2/a)^2}{\Xi^{-2}\rho^2\Delta_\theta\Delta_r\sin^2\theta}\left(\frac{\partial}{\partial t}+\frac{ieQr}{\rho^2\Xi}\right)^2 \\
&\quad - 2\frac{\Delta_\theta(r^2+a^2)a\sin^2\theta}{\Xi^{-2}\rho^2\Delta_\theta\Delta_r\sin^2\theta}\left(\frac{\partial}{\partial t}+\frac{ieQr}{\rho^2\Xi}\right)\left(\frac{\partial}{\partial\phi}-\frac{ieQra\sin^2\theta}{\rho^2\Xi}\right) \\
&\quad + 2\frac{\Delta_r(a-(n-a\cos\theta)^2/a)}{\Xi^{-2}\rho^2\Delta_\theta\Delta_r\sin^2\theta}\left(\frac{\partial}{\partial t}+\frac{ieQr}{\rho^2\Xi}\right)\left(\frac{\partial}{\partial\phi}-\frac{ieQra\sin^2\theta}{\rho^2\Xi}\right) \\
&\quad + \frac{\Delta_r-\Delta_\theta a^2\sin^2\theta}{\Xi^{-2}\rho^2\Delta_\theta\Delta_r\sin^2\theta}\left(\frac{\partial}{\partial\phi}-\frac{ieQra\sin^2\theta}{\rho^2\Xi}\right)^2 \\
&\quad + \frac{1}{\rho^2}\frac{\partial}{\partial r}\left(\Delta_r\frac{\partial}{\partial_r}\right)+\frac{1}{\sin\theta\rho^2}\frac{\partial}{\partial\theta}\left(\Delta_\theta\sin\theta\frac{\partial}{\partial\theta}\right)\Bigg]\Phi,
\end{aligned} \tag{7}$$

Φ could be decompounded as $\Phi = \Sigma_{l,m}\phi(t,r)Y_{lm}(\theta,\varphi)$. By transforming to the "tortoise" coordinate transformation defined by $dr_*/dr = (r^2+a^2)\Xi/\Delta_r^2 \equiv f^{-1}(r)$, we can get the simplified action

$$S[\phi] = \frac{(r^2+a^2)}{2\Xi}\sum_{l,m}\int dtdr\phi_{l,m}^*\left[\partial_r f(r)\partial_r - \frac{1}{f(r)}\left(\partial_t + \frac{ima}{(r^2+a^2)} + \frac{ieQr}{(r^2+a^2)\Xi}\right)^2\right]\phi_{l,m}. \tag{8}$$

Here, we let

$$\Omega = \frac{a}{r^2+a^2}, \qquad \Psi = \frac{Qr}{(r^2+a^2)\Xi}, \qquad A_t(r) = -m\Omega - e\Psi \tag{9}$$

using the simplified $(1+1)$-dimensional theory, near the horizons, we can conveniently deal with the Hawking radiation, as will be shown in the following discussions.

3 The Hawking Radiation at EH

We will research Hawking radiation from the EH in this part. Above all, let's investigate the gauge anomaly. When researching on the gauge anomaly, we should consider charge and rotation respectively. In the region $r_h < r < r_h + \varepsilon$, we omit the ingoing modes. The consistent form of $d = 2$ Abelian anomaly are

$$\nabla_\mu J^\mu_{(h)e} = \pm e \frac{\varepsilon^{\mu\nu}}{4\pi\sqrt{-g}} \partial_\mu A_\nu, \tag{10}$$

$$\nabla_\mu J^\mu_{(h)m} = \pm m \frac{\varepsilon^{\mu\nu}}{4\pi\sqrt{-g}} \partial_\mu A_\nu \tag{11}$$

where $\varepsilon^{10} = 1$ and $+$ $(-)$ corresponds to left (right)-handed field. If we let

$$J^\mu_{(h)} \equiv \frac{J^\mu_{(h)e}}{e} \equiv \frac{J^\mu_{(h)m}}{m} \tag{12}$$

the uniform form is

$$\nabla_\mu J^\mu_{(h)} = \pm \frac{\varepsilon^{\mu\nu}}{4\pi\sqrt{-g}} \partial_\mu A_\nu. \tag{13}$$

So we get

$$\partial_r J^r_{(h)} = \frac{1}{4\pi} \partial_r A_t. \tag{14}$$

But, in $r_h + \varepsilon < r < r_c$, $J^r_{(o)}$ satisfies

$$\partial_r J^r_{(o)} = 0 \tag{15}$$

where

$$J^\mu_{(o)} \equiv \frac{J^r_{(o)e}}{e} \equiv \frac{J^r_{(o)m}}{m} \tag{16}$$

from (14) and (15) we can obtain

$$J^r_{(o)} = c_0, \quad r_h + \varepsilon < r < r_c, \tag{17}$$

$$J^r_{(h)} = c_h + \frac{1}{4\pi} \left(A_t(r) - A_t(r_h) \right), \quad r_h < r < r_h + \varepsilon \tag{18}$$

where

$$A_t(r_h) = -\frac{am}{r_h^2 + a^2} - \frac{Qer_h \Xi^{-1}}{r_h^2 + a^2}. \tag{19}$$

We need to get the values of c_o and c_h using the effective action and obtain the gauge current, so we let

$$J^r = J^r_{(o)} \Theta_+(r) + J^r_{(h)} H(r) \tag{20}$$

where

$$\Theta_+(r) = \Theta_+(r - r_h - \varepsilon), \qquad H(r) = 1 - \Theta_+(r), \tag{21}$$

$$\Theta(r - r_h - \varepsilon) = \begin{cases} 1, & r_h + \varepsilon < r < r_c, \\ 0, & r_h < r < r_h + \varepsilon. \end{cases} \tag{22}$$

Then the variation of effective action is

$$-\delta W=\int dtdr\lambda\nabla_\mu J^r=\int dtdr\lambda\left[\left(J^r_{(o)}-J^r_{(h)}+\frac{A_t}{4\pi}\right)\delta\left(r-r_h-\varepsilon\right)+\partial_r\left(\frac{A_t}{4\pi}H\right)\right] \tag{23}$$

where λ is the general coordinate transformation parameter, and from the integral, we get

$$J^r_{(o)}=J^r_{(h)}-\frac{A_t\left(r\right)}{4\pi}. \tag{24}$$

So, at EH

$$c^r_{(o)}=c^r_{(h)}-\frac{A_t\left(r_h\right)}{4\pi}. \tag{25}$$

The covariant current is

$$\tilde{J}^\mu=J^\mu\pm\frac{\varepsilon^{\mu\nu}}{4\pi\sqrt{-g}}A_\nu. \tag{26}$$

The current satisfies

$$\nabla_\mu\tilde{J}^\mu=J^\mu\pm\frac{\varepsilon_{\mu\nu}}{4\pi}F_{\mu\nu}. \tag{27}$$

In the $(1+1)$-dimensional anomalous theory, the covariant current that is introduced for canceling anomalies is

$$\tilde{J}^r=J^r+\frac{1}{4\pi}A_t\left(r_h\right)H\left(r\right). \tag{28}$$

So, from (17), (18), (19), (25) and (28), we can obtain

$$c_o=-\frac{A_t\left(r_h\right)}{2\pi}. \tag{29}$$

This is just the gauge current that we want to get.

Now, we research the flux of the energy-momentum tensor current near event horizon via gravitational anomaly point of view. Similarly, we just consider the outgoing modes. Near the EH, we define

$$T^\mu_\nu=T^\mu_{\nu(o)}\Theta_+\left(r\right)+T^\mu_{\nu(h)}H\left(r\right). \tag{30}$$

From the Ward identical equation

$$\nabla_\mu T^\mu_\nu=F_{\mu\nu}J^\mu+A_\nu\nabla_\mu J_\mu+\nabla_\mu N^\mu_\nu \tag{31}$$

where

$$N^\mu_\nu=\frac{1}{96\pi}\varepsilon^{\beta\mu}\partial_\alpha\Gamma^\alpha_{\nu\beta}. \tag{32}$$

From 2-dimension Levi-Civita tensor, if we just consider the case $\nu=t$, we can get

$$N^r_t=\frac{1}{192\pi}(f'^2+ff''). \tag{33}$$

In $r_h+\varepsilon<r<r_c$, $T^r_{t(o)}$ satisfies

$$\partial_r T^r_t\left(o\right)=c_o\partial_r A_t. \tag{34}$$

While in region $r_h < r < r_h + \varepsilon$, $T^r_{t(h)}$ will satisfy the anomalous equation

$$\partial_r T^r_{t(h)} = J^r_{(h)} \partial_r A_t + A_t \partial_r J^r_{(h)} + \partial_r N^r_t . \tag{35}$$

Variation of the action can be expressed as

$$-\delta W = \int dt dr \lambda^\nu \nabla_\mu T^\mu_\nu = \int dt dr \lambda^t \left[\partial_r \left[\left(\frac{A_t^2}{4\pi} + N^r_t \right) H \right] + \left(T^r_{t(o)} - T^r_{t(h)} + \frac{A_t^2}{4\pi} + N^r_t \right) \delta \left(r - r_h - \varepsilon \right) + c_o \partial_r A_t \right]. \tag{36}$$

The second term is the quantum boundary effect, and the third term is the classical effect of the background electromagnetic field for current flow, so we set, near the EH, the covariant energy-momentum tensor is

$$\tilde{T}^r_{t(h)} = T^r_{t(h)} + \frac{1}{192\pi} (f f'' - 2 f'^2) \tag{37}$$

which satisfies [23]

$$\nabla_\mu \tilde{T}^\mu_\nu = \frac{1}{96\pi \sqrt{-g}} \varepsilon_{\mu\nu} \partial^\mu R. \tag{38}$$

So from (34), (35) and (36), at the EH , we obtain

$$k_o = k_h + \frac{A_t^2 (r_h)}{4\pi} - N^r_t (r_h) \tag{39}$$

where

$$k_o = T^r_{t(o)} - c_o A_t (r_h) , \tag{40}$$

$$k_h = T^r_{t(h)} - \int_{r_h}^{r} dr \partial_r \left[c_o A_t (r_h) + \frac{A_t^2}{4\pi} - N^r_t \right]. \tag{41}$$

The energy-momentum tensor current can express as

$$k_o = \frac{A_t^2 (r_h)}{4\pi} + \frac{\pi}{12} T_h^2 \tag{42}$$

where $T_h = \partial_r f(r) / (4\pi) |_{r=r_h}$ is known as the Hawking temperature of the black hole.

For fermions, the Hawking radiation spectrum from EH of the black hole is

$$N_{\pm m} (\omega) = 1 / \left[\exp \left(\frac{\omega \pm A_t (r_h)}{T_h} \right) + 1 \right]. \tag{43}$$

We can obtain the gauge current and the energy-momentum tensor current fluxes from the formulae of quantum field theory

$$F_J = \frac{1}{2\pi} \int_0^\infty [N_+ (\omega) - N_- (\omega)] d\omega = -\frac{A_t (r_h)}{2\pi}. \tag{44}$$

Corresponding to (16)

$$F_J \equiv \frac{F_J^m}{m} \equiv \frac{F_J^e}{e} \tag{45}$$

where

$$F_J^e = \frac{e}{2\pi}\int_0^\infty [N_+(\omega) - N_-(\omega)]d\omega = -e\frac{A_t(r_h)}{2\pi}, \tag{46}$$

$$F_J^m = \frac{m}{2\pi}\int_0^\infty [N_+(\omega) - N_-(\omega)]d\omega = -m\frac{A_t(r_h)}{2\pi}, \tag{47}$$

and

$$F_T = \frac{1}{2\pi}\int_0^\infty \omega[N_+(\omega) + N_-(\omega)]d\omega = \frac{A_t^2(r_h)}{4\pi} + \frac{\pi}{12}T_h^2. \tag{48}$$

From the known Hawking radiation spectrum and corresponding Hawking radiation current, we obtain (44), which accords with (29). We also obtain (48) from the known Hawking radiation result, and find it accords with (42). The results show that we have accomplished the research of pure thermal spectrum from the event horizon of NUT-Kerr-Newman de Sitter black hole.

4 The Hawking Radiation from CH

Near the CH, what we should consider are ingoing modes. We introduce the modes inside the CH, in $rc - \varepsilon < r < rc$, the modes need to satisfy the anomalous equation

$$\partial_r J_{(c)}^r = \frac{-\partial_r A_t}{4\pi} \tag{49}$$

where

$$J_{(c)}^r \equiv \frac{J_{(c)e}^r}{e} \equiv \frac{J_{(c)m}^r}{m}. \tag{50}$$

The following two equations should be satisfied

$$\partial_r J_{(c)e}^r = \frac{-e}{4\pi}\partial_r A_t, \tag{51}$$

$$\partial_r J_{(c)m}^r = \frac{-m}{4\pi}\partial_r A_t. \tag{52}$$

But in $r_h < r < r_c - \varepsilon$, there is no anomaly, so the equation is

$$\partial_r J_{(o)}^r = 0 \tag{53}$$

where

$$J_{(o)}^r \equiv \frac{J_{(co)e}^r}{e} \equiv \frac{J_{(o)m}^r}{m}. \tag{54}$$

New, from the formulae (51), (52) and (53), we can obtain

$$J_{(o)}^r = h_0, \quad r_h < r < r_c - \varepsilon, \tag{55}$$

$$J_{(c)}^r = h_c - \frac{1}{4\pi}(A_t(r) - A_t(r_c)), \quad r_h - \varepsilon < r < r_c \tag{56}$$

where

$$A_t(r_c) = -\frac{am}{r_c^2+a^2} - \frac{Qer_c\Xi^{-1}}{r_c^2+a^2}. \tag{57}$$

Similarly, we let

$$J^r = J^r_{(o)}\Theta_- + J^r_{(c)}C \tag{58}$$

where

$$\Theta_-(r) = \Theta(r_c - r - \varepsilon), \qquad C(r) = 1 - \Theta_+(r). \tag{59}$$

The simplified effective action variation

$$-\delta W = \int dtdr\lambda\nabla_\mu J^\nu = \int dtdr\lambda\left[\left(J^r_{(c)} - J^r_{(o)} + \frac{A_t}{4\pi}\right)\delta(r_c - r - \varepsilon) + \partial_r\left(\frac{A_t}{4\pi}C\right)\right], \tag{60}$$

$\frac{A_t}{4\pi}C$ is the boundary term of quantum effect, then we have

$$J^r_{(o)} = J^r_{(c)} - \frac{A_t}{4\pi}. \tag{61}$$

We introduce a covariant current to cancel anomalous near the CH

$$\tilde{J}^r = J^r - \frac{A_t(r_c)C(r)}{4\pi} \tag{62}$$

corresponding to (29), from (55), (56), (61) and (62), we obtain

$$h_o = \frac{A_t(r_c)}{2\pi}. \tag{63}$$

Researching the energy-momentum tensor current, and omitting outgoing modes near the CH of black hole, let's define

$$T^\mu_\nu = T^\mu_{\nu(o)}\Theta_-(r) + T^\mu_{\nu(c)}C(r). \tag{64}$$

In the region $r_h + \varepsilon < r < r_c$, $T^r_{t(o)}$ satisfies

$$\partial_r T^r_{t(o)} = h_o\partial_r A_t. \tag{65}$$

In $r_c - \varepsilon < r < r_c$, the anomalous equation of $T^r_{t(c)}$ is

$$\partial_r T^r_{t(c)} = J^r_{(c)}\partial_r A_t + A_t\partial_r J^r_{(c)} - \partial_r N^t_r. \tag{66}$$

The variation of effective action is

$$\begin{aligned} -\delta W = \int dtdr\lambda^\nu\nabla_\mu T^\mu_\nu \int = dtdr\lambda^t\Bigg[\bigg(\left(\frac{A_t^2}{4\pi} + N^r_t\right)C\bigg) \\ \times\left(T^r_{t(c)} - T^r_{t(o)} + \frac{A_t^2}{4\pi} + N^r_t\right)\delta(r_c - r + \varepsilon) + \partial_r + h_o\partial_r A_t\Bigg]. \end{aligned} \tag{67}$$

The third term is the classical effect of the background electromagnetic field of the current and the second term is the quantum boundary effect. Finally, we can define, at CH, the covariant energy-momentum tensor current flux

$$\tilde{T}^r_{t(c)} = T^r_{t(c)} - \frac{1}{192\pi}(ff'' - 2f'^2). \tag{68}$$

Paying attention to (65), (66), (67) and (68), we get

$$n_o = n_c - \frac{A_t^2(r_c)}{4\pi} + N_t^r(r_c) \tag{69}$$

where

$$n_o = T^r_{t(o)} - h_o A_t(r_c), \tag{70}$$

$$n_c = T^r_{t(c)} - \int_{r_c}^{r} dr\,\partial_r\left[h_o A_t - \frac{A_t^2}{4\pi} - N_t^r\right]. \tag{71}$$

The energy-momentum tensor current is

$$n_{(o)} = -\frac{A_t^2(r_c)}{4\pi} - \frac{\pi}{12}T_c^2 \tag{72}$$

where, $T_c = -\partial_r f(r)/(4\pi)|_{r=r_c}$ is the temperature of black hole.

For fermions, the Planckian distribution of the Hawking radiation from the CH of the black hole is

$$N_{\pm m}(\omega) = -1\Big/\left[\exp\left(\frac{\omega \pm A_t(r_c)}{T_c}\right) + 1\right]. \tag{73}$$

We obtain the gauge current from the formulae of quantum field theory

$$F_J = \frac{1}{2\pi}\int_0^\infty [N_+(\omega) - N_-(\omega)]d\omega = \frac{A_t(r_c)}{2\pi} \tag{74}$$

where

$$F_J \equiv \frac{F_J^m}{m} \equiv \frac{F_J^e}{e}. \tag{75}$$

Corresponding to (45) and (46), we have

$$F_J^e = \frac{e}{2\pi}\int_0^\infty [N_+(\omega) - N_-(\omega)]d\omega = e\frac{A_t(r_c)}{2\pi}, \tag{76}$$

$$F_J^m = \frac{m}{2\pi}\int_0^\infty [N_+(\omega) - N_-(\omega)]d\omega = m\frac{A_t(r_c)}{2\pi}. \tag{77}$$

And the energy-momentum tensor current is

$$F_T = \frac{1}{2\pi}\int_0^\infty \omega[N_+(\omega) + N_-(\omega)]d\omega = -\frac{A_t^2(r_c)}{4\pi} - \frac{\pi}{12}T_c^2. \tag{78}$$

From (74) and (78), we can know that (63) and (72) are the results of Hawking radiation.

5 Conclusions

From the results of this paper, if $n = 0$ and $\Xi = 1$, we get the conclusion of Kerr-Newman black hole. When $\Xi = 1$, $Q = 0$, and $n = 0$ we can get the Hawking radiation of Kerr black hole.

According to the theory of black hole radiation, near the horizon of the black hole, there is quantum vacuum fluctuation effect, so virtual particle pairs maybe appear. The negative energy particle enter the EH (or exit the CH) and become ingoing modes (or outgoing modes), but the positive energy particle would emit and become outgoing modes (or ingoing modes). The latter is no other than what we want to research, so we just consider the outgoing mode at the EH (or outgoing modes at the CH). From the anomalous equations, we can obtain the right conclusions finally.

Acknowledgement The work is supported by National Natural Science Foundation of People's Republic of China (No. 10773008).

References

1. Wu, S.Q., Cai, X.: Non-existence of new quantum ergosphere effect of a Vaidya-type black hole. Mod. Phys. Lett. A **16**, 1549 (2001)
2. Parikh, M.K.: New coordinates for de Sitter space and de Sitter radiation. Phys. Lett. B **546**, 189 (2002)
3. Ren, J., Zhao, Z., Gao, C.J.: Hawking radiation via tunnelling from arbitrarily dimensional Schwarzschild black holes. Chin. Phys. Lett. **22**, 2489 (2005)
4. Ren, J., Zhang, J.Y., Zhao, Z.: Tunnelling effect and Hawking radiation from a Vaidya black hole. Chin. Phys. Lett. **23**, 2019 (2006)
5. Parikh, M.K., Wilczek, F.: Hawking radiation as tunneling. Phys. Rev. Lett. **85**, 5042 (2000)
6. Liu, M.Q., Yang, S.Z.: Tunneling radiation of the charged particles for charged spherical black hole in VGM. Int. J. Theor. Phys. **46**, 63 (2007)
7. Han, Y.W.: Massive particles' tunnelling radiation from the black hole with a mass-quadruple moment. Chin. Phys. **16**, 0923 (2007)
8. Yang, S.Z., Jiang, Q.Q.: Research on Hawking radiation as tunneling from Schwarzschild-anti-de Sitter black hole. Int. J. Theor. Phys. **46**, 2138 (2007)
9. Wang, X.X., Yang, S.Z.: Casimir effect under two dimensional black hole spacetime background with global monopole. Int. J. Theor. Phys. **46**, 1797 (2007)
10. Zhang, J.Y., Zhao, Z.: New coordinates for the evaporating Vaidya black hole. Chin. Phys. Lett. **23**, 1099 (2006)
11. Hawking, S.W.: Black hole explosions? Nature **248**, 30 (1974)
12. Robinson, S.P., Frank, W.: Relationship between Hawking radiation and gravitational anomalies. Phys. Rev. Lett. **95**, 011303 (2005). gr-qc/0502074
13. Jiang, Q.Q., Wu, S.Q., Cai, X.: Hawking radiation as tunneling from the Kerr and Kerr-Newman black holes. Phys. Rev. D **73**, 064003 (2006)
14. Jiang, Q.Q., Wu, S.Q., Cai, X.: Hawking radiation from dilatonic black holes via anomalies. Phys. Rev. D **75**, 064029 (2007)
15. Lin, K., Zeng, X.X., Yang, S.Z.: Hawking radiation from topological Kerr-Anti-de Sitter black hole with one rotational parameter via covariant anomalies. Chin. Phys. Lett. **25**, 390 (2008)
16. Zeng, X.X., Chen, D.Y., Yang, S.Z.: Quantum anomaly at horizon and Hawking radiation from higher dimensional Reissner-Nordström de Sitter black hole. Gen. Relativ. Gravit. (2007). DOI:10.1007/s10714-007-0543-y
17. Jiang, Q.Q., Wu, S.Q.: Hawking radiation from rotating black holes in anti-de Sitter spaces via gauge and gravitational anomalies. Phys. Lett. B **647**, 200 (2007). hep-th/0701002
18. Wu, S.Q., Peng, J.J.: Anomalies and Hawking radiation from the Reissner-Nordström black hole with a global monopole. Class. Quantum Gravity **24**, 5123 (2007). arXiv:0706.0983
19. Iso, S., Umetsu, H., Wilczek, F.: Anomalies, Hawking radiations and regularity in rotating black holes. Phys. Rev. D **74**, 044017 (2006). hep-th/0606018
20. Murata, K., Sod, J.: Hawking radiation from rotating black holes and gravitational anomalies. Phys. Rev. D **74**, 044018 (2006). hep-th/0606069

21. Ahmed, M.: Hawking radiation of Dirac particles in the hot NUT-Kerr-Newman spacetime. Phys. Lett. B **258**, 318 (1991)
22. Ge, X.H., Shen, Y.G.: Entropy in the NUT Kerr Newman black holes due to an arbitrary spin field. Class. Quantum Gravity **20**, 3593 (2003)
23. Bertlmann, R., Kohlprath, E.: Two-dimensional gravitational anomalies, Schwinger terms and dispersion relations. Ann. Phys. (NY) **288**, 137 (2001). hep-th/0011067